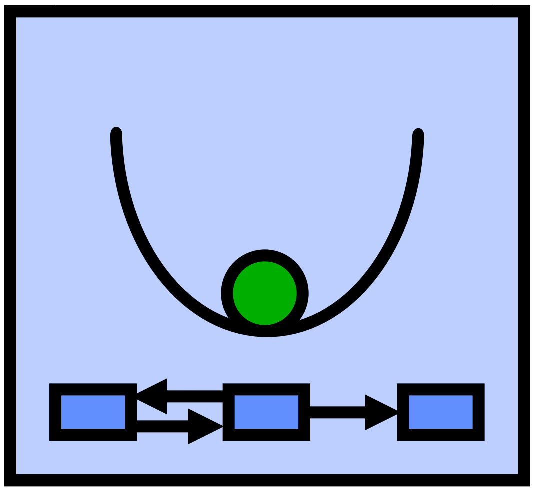
Stability Analysis in Communication Networks

Seminar-Presentation

Presentation 30.06.04



- **Fluid Models**
- **Stability Methods**
- **Conclusion**

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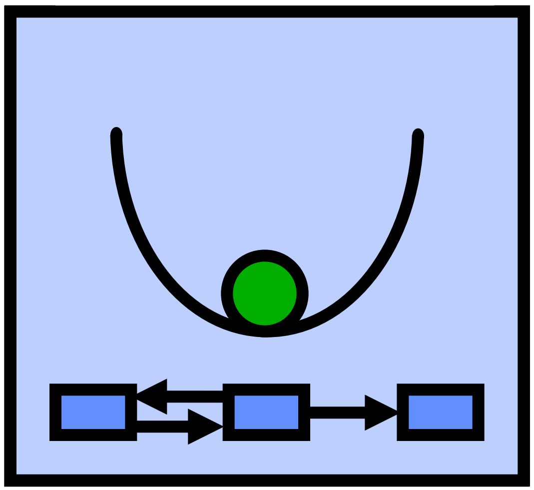
Topic Overview



- **Fluid Models**
- **Stability Methods**
- **Conclusion**

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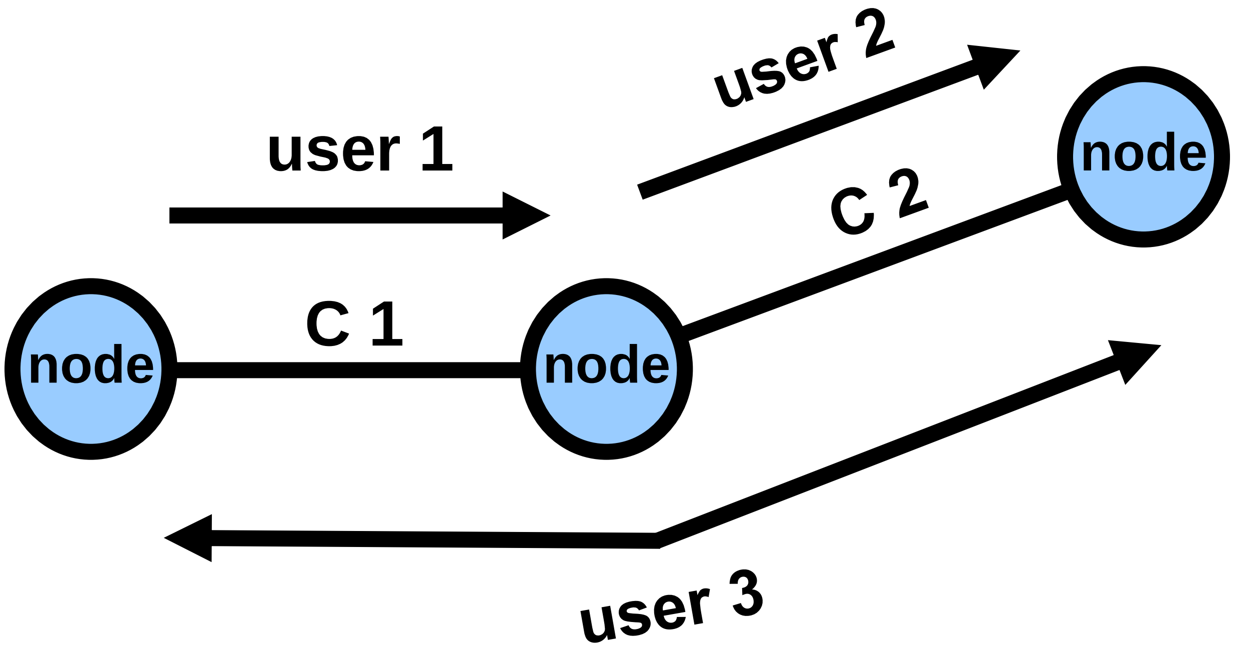
Fluid Models – Working groups

- Fluid Models, Game Theory, Randomized Algorithms
  - Tamer Başar, Tansu Alpcan, R. Srikant, Decision and Control Laboratory, University of Illinois, Champaign-Urbana
- Fluid Models, TCP/IP
  - Yong Liu, Francesco Lo Presti, Vishal Misra, Don Towsley and Yu Gu, University of Massachusetts, Computer Science Department, Amherst
    - http://www-net.cs.umass.edu/fluid/

Fluid Models - Introduction 1

- Arbitrary network structure



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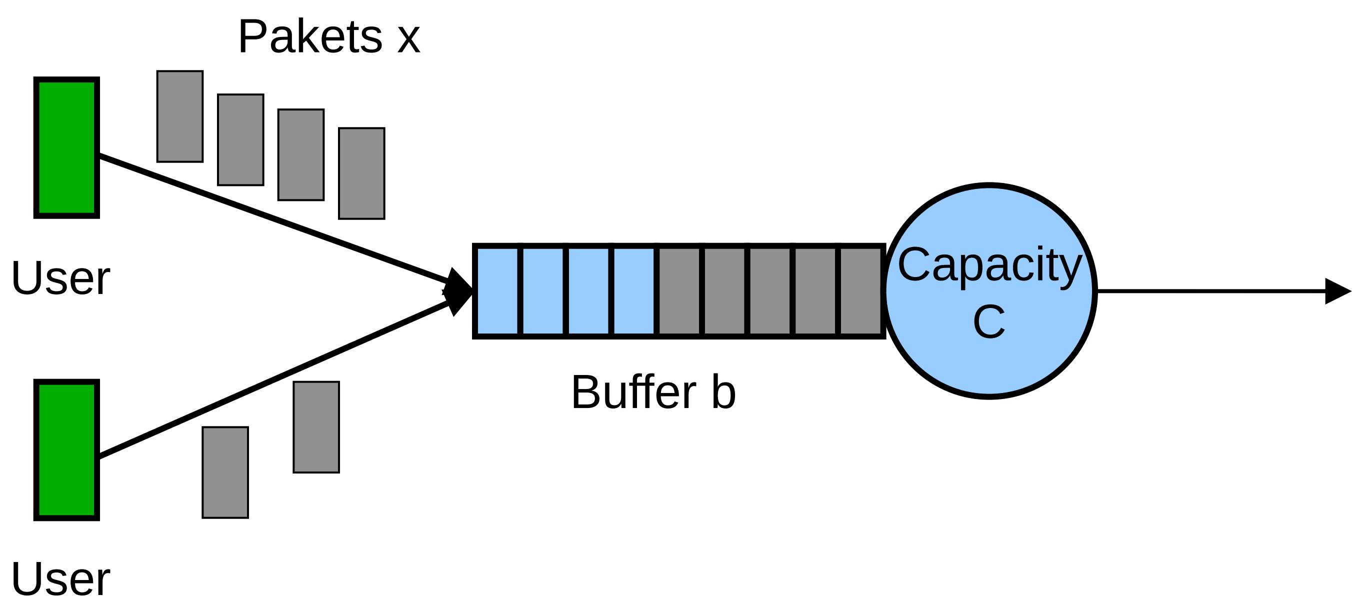
Fluid Models - Introduction 1



- Data packets and buffer level are described through discrete values

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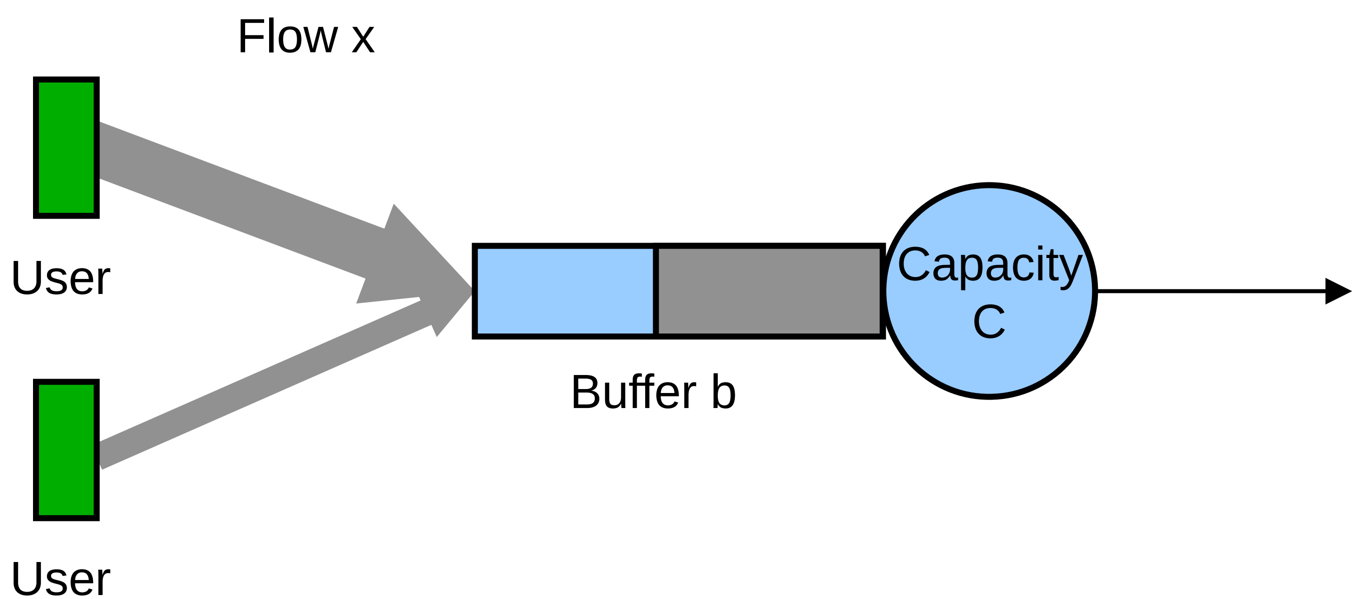
Fluid Models - Introduction 2



- Now represented through continuous values
- Real world analogy: Flow of a fluid

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Fluid Models – System Dynamics 1

$$\dot{b}_l(t) = \begin{cases} \left[\bar{x}_l - C_l\right]^-, & b_l(t) = b_{l,\max} \\ \bar{x}_l - C_l, & 0 < b_l(t) < b_{l,\max} \\ \left[\bar{x}_l - C_l\right]^+, & b_l(t) = 0 \end{cases}$$

- Change of buffer level at node *l*
  = incoming flow – capacity of the outgoing link

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Fluid Models – System Dynamics 2

$$\dot{d}_l(x,t) = \begin{cases} \left[\frac{1}{C_l} \left(\bar{x}_l - C_l \right) \right]^-, & d_l(t) = d_{l,\max} \\ \frac{1}{C_l} \left(\bar{x}_l - C_l \right), & 0 < d_l(t) < d_{l,\max} \\ \left[\frac{1}{C_l} \left(\bar{x}_l - C_l \right) \right]^+, & d_l(t) = 0 \end{cases}$$

- Change of delay at node *l* (FCFS) = change of buffer level / capacity of the outgoing link

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Fluid Models – System Dynamics 3

Price → α_i; **Delay over all links used by user *i*** → $D_i(t)$; **Utility** → $U_i(x_i)$

$$J_i(x,t) = \alpha_i D_i(t) x_i - U_i(x_i)$$

$$\dot{x}_i = -\partial J_i(x,t) / \partial x_i$$

- Adaptation of flow rate in accordance with cost function *J*
- Game theory: Unique point where the costs per player are minimized and every change of flow increases the cost of the player (Nash Equilibrium)

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Fluid Models – System Dynamics 4

- The system dynamics are described through the following differential equations.
  - ignoring the effects of boundaries

$$\dot{x}_i(t) = \frac{dU_i(x_i)}{dx_i} - \alpha_i D_i(x,t)$$

$$\dot{d}_l(t) = \frac{\bar{x}_l}{C_l} - 1$$

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Fluid Models – TCP System Dynamics 1

- Model of the congestion avoidance part of the TCP slow-start algorithm
  - Bottleneck situation, without timeout mechanism
- TCP increases window size by one every RTT (linear)
- If a congestion occurs the window size will be reduced to half of the original size

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)}{2} \cdot \underbrace{\frac{W(t-R(t))}{R(t-R(t))} \cdot p(t-R(t))}_{\lambda_C}$$

Fluid Models – TCP System Dynamics 2

- Change of queue length depends on the capacity of the link and the incoming flow

$$\dot{q}(t) = \underbrace{N(t) \cdot \frac{W(t)}{R(t)}} - C$$

Incoming flow = number of connections (N) and sent data per RTT (W/R)

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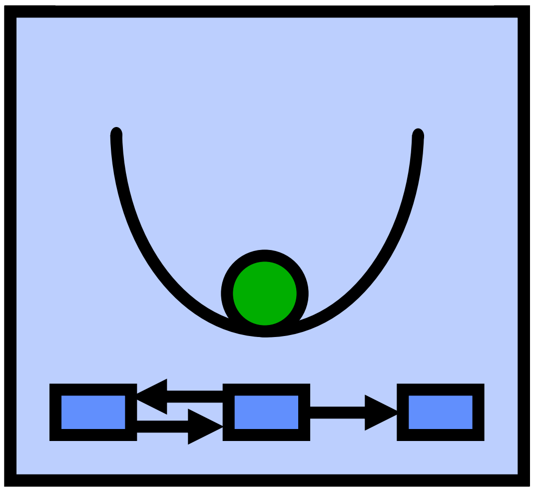

- Fluid Models
- **Stability Methods**
- Conclusion

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Stability Methods – Introduction

- Linearization of the nonlinear model around the operating point (OP) to investigate its behavior
  - i.e. Taylor series expansion
  - OP: gradient is zero
- Prediction of stability:
  - energy like Lyapunov function
  - Monte Carlo Simulation
  - Laplace Transformation: open/closed loop investigation

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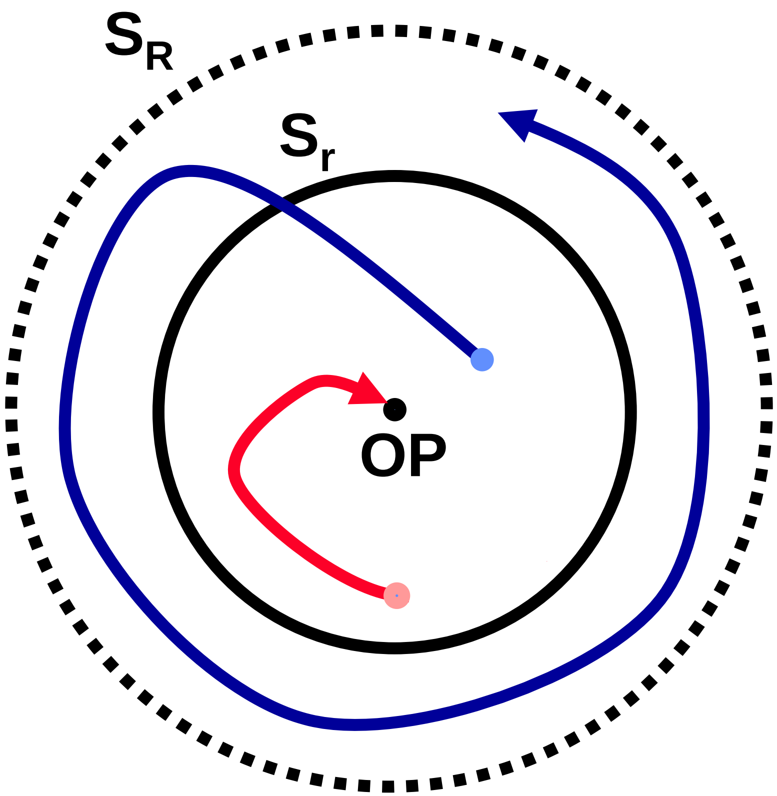
Stability Methods – Lyapunov Function 1



- Lyapunov stable: Trajectory of the System never leaves area S_R if it starts within S_r
  - For any given R>0 exist at least one r>0
- Asymptotically stable: If Lyapunov stable and trajectory is heading towards OP for t->∞

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Stability Methods – Lyapunov Function 2

- Analogy: Energy of a system.
  - If a system perpetually loses energy it is asymptotically stable
    - As long as there are no energy inputs
- Generalization: Function V which describes how much energy is in the system
  - Continuously differentiable
  - Positive definite
  - First partial derivative negative (semi)definite
    - Negative definite if asymptotically stable

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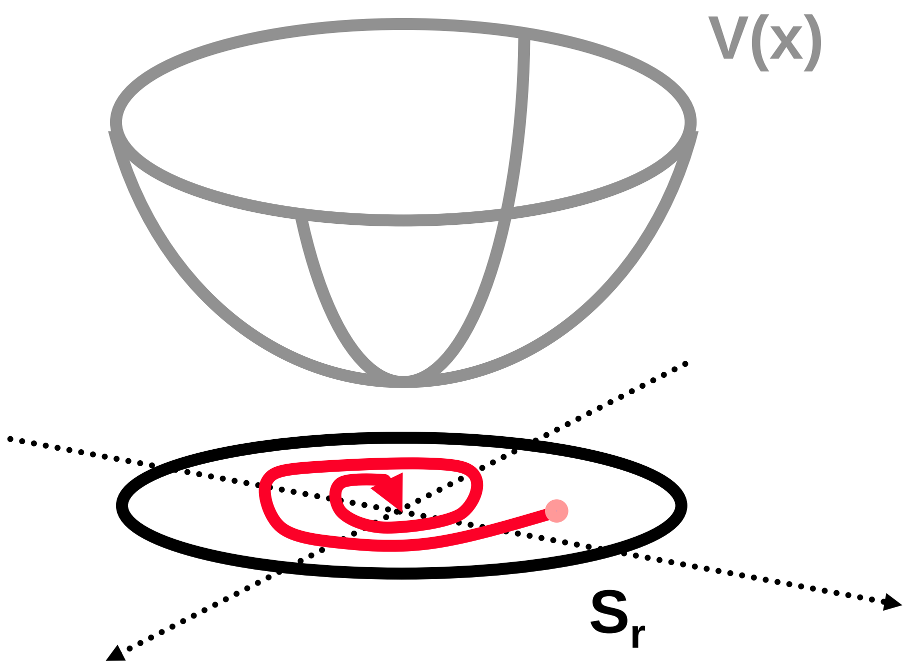
Stability Methods – Lyapunov Function 3



– Positive definite if V(0)=0 and V(a)>0 for a!=0
  - Energy is in the system if not in OP point
– negative (semi)definite if V'(0)=0 and V'(a)<=0 for all a
  - System loses Energy

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Stability Methods – Lyapunov Function Example

- The first example system was described through x' and d'
- Amount of data in the flow and the buffer which deviates from the OP(x_L,d_L) is the energy in the system
- V'(x_L,d_L) is negative semidefinite if the utility function U is strictly increasing and concave and the RTT is smaller than a certain computable value

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Stability Methods – Monte Carlo Simulation 1

- Randomized algorithms are a useful non-analytical tool for investigating stability and robustness of a system
- Monte Carlo methods use random sampling
  - Discover whether the members of a set N have a certain property by randomly choosing a member from that set and testing it
  - Repeating this process obtains a probabilistic estimate on whether the set does have that property

$$\hat{p} = \frac{N_{Stable}}{N_{Total}}$$

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Stability Methods – Monte Carlo Simulation 2

- Discretized version of the first example system model described in matrix form

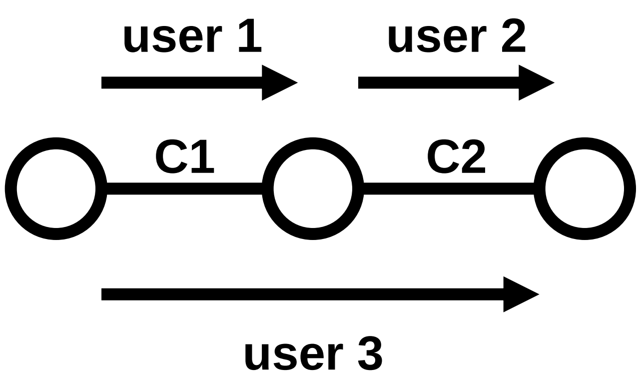
$$\begin{pmatrix} x_L(t+1) \\ d_L(t+1) \end{pmatrix} = G(\alpha, u, C) \begin{pmatrix} x_L(t) \\ d_L(t) \end{pmatrix}$$

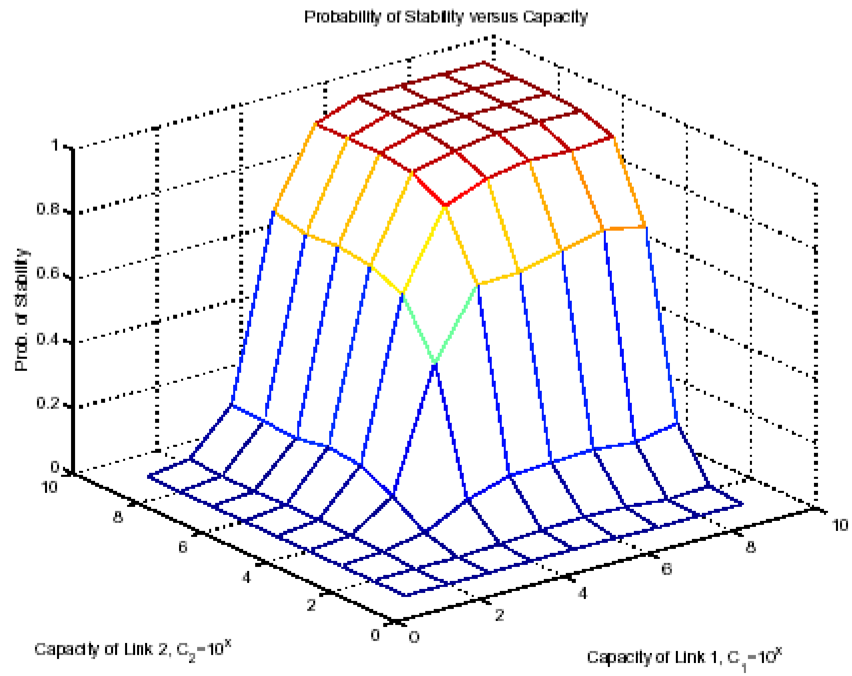
- The discretized system is stable if the absolute values of the eigenvalues of G are less than 1
- Analyze probability of stability for a given parameter space (M-user)

$$p = \left[\alpha_1, \ldots, \alpha_M, u_1, \ldots, u_M, C_1, \ldots, C_L\right]$$

Stability Methods – Monte Carlo Simulation Example

- One Result: Probability of stability increases with increasing capacity of the links
  - 3 Users, 2 Links, Parameter ranges $0<u_i<1$, $0<\alpha_i<1000$, $10^2<C_l<10^6$





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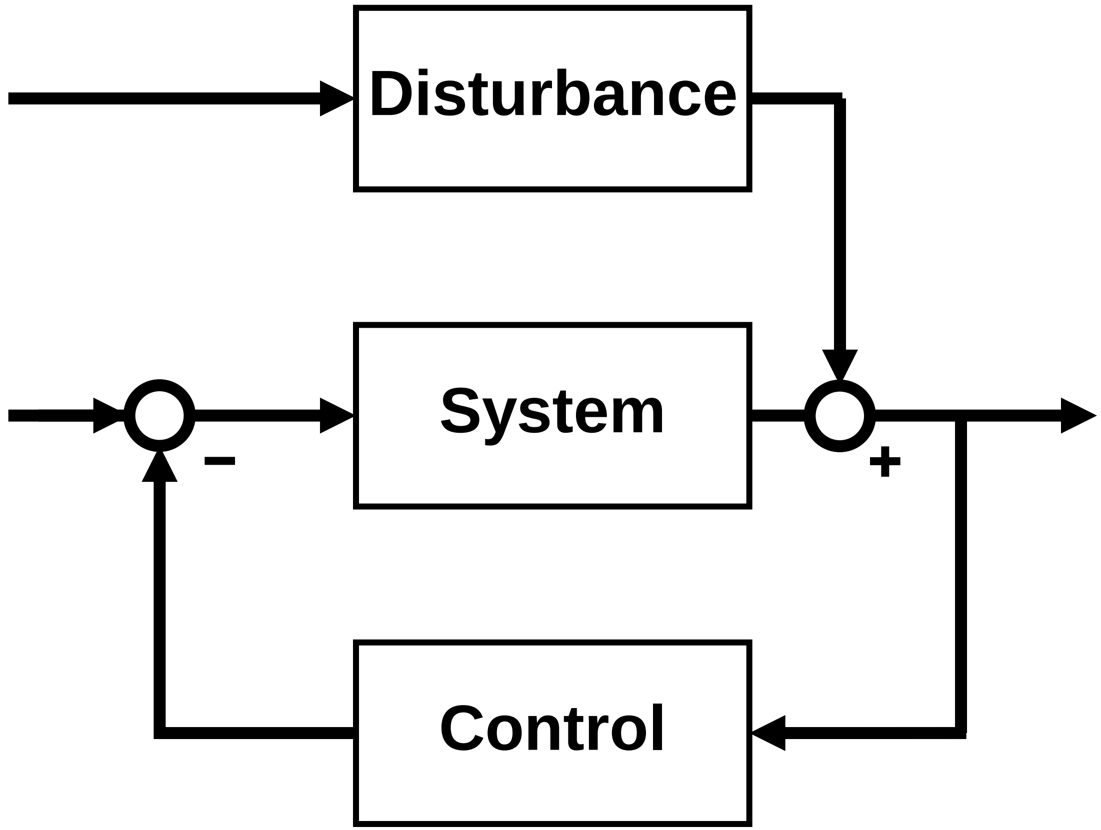
Stability Methods – open/closed loop investigations



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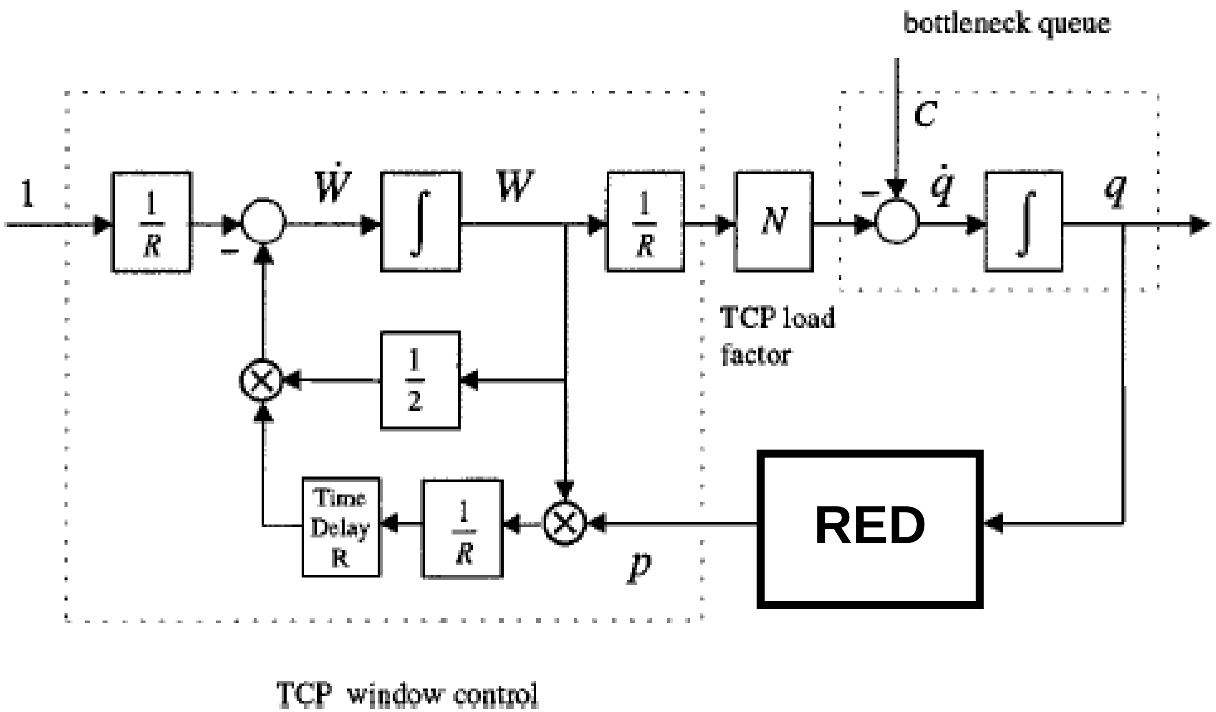
Stability Methods – TCP System Dynamics



- W= TCP window size
- q= Queue length
- R= Round trip time
- C= Link capacity
- N= Number TCP connections
- p= probability of marked packets
- RED= Random Early Detection

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Stability Methods – open/closed loop investigations

- Transformation into Laplace space eases solving of integral and differential equations

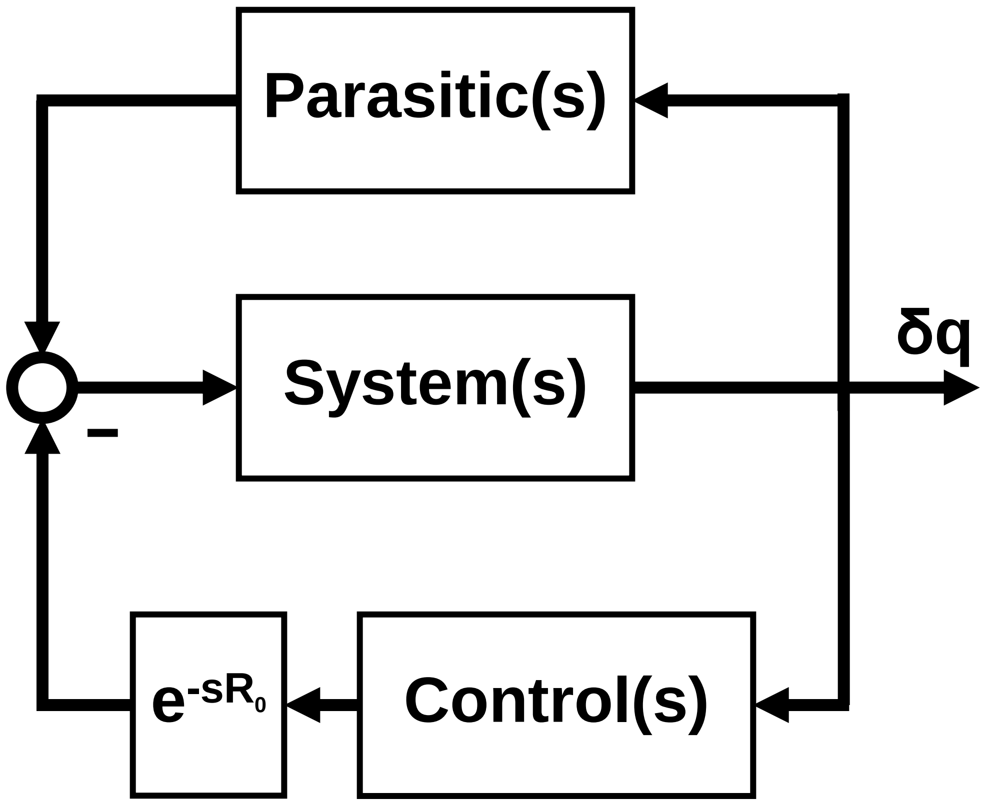
$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n f(s)$$

$$L\left[\int f(t)dt\right] = \frac{1}{s} f(s)$$

$$s = \sigma + j\omega$$

Stability Methods – open/closed loop investigations

- After linearization and transformation into Laplace space



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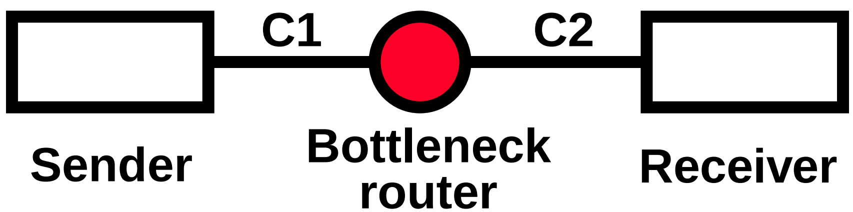
Stability Methods – open/closed loop investigations

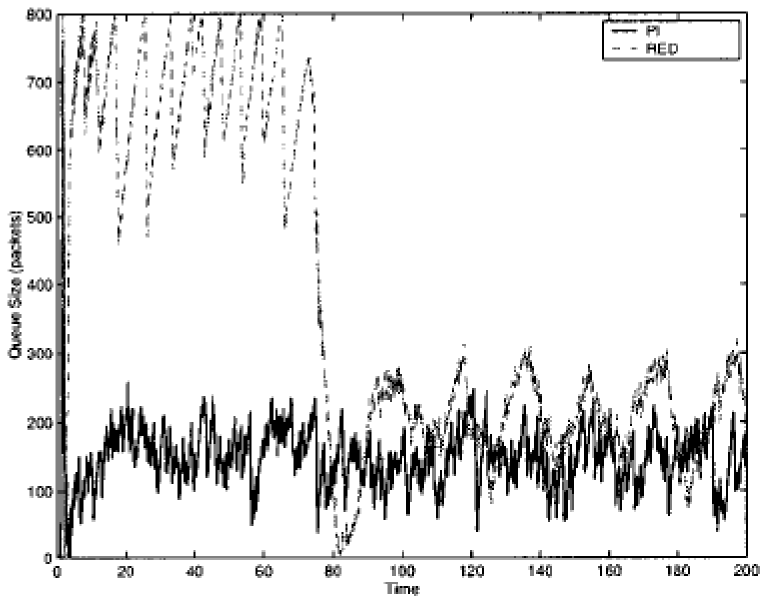
- Instead of using the RED control law a Proportional / Integral PI-Controller was used
- PI-Controller was tuned to the investigated system dynamics of TCP



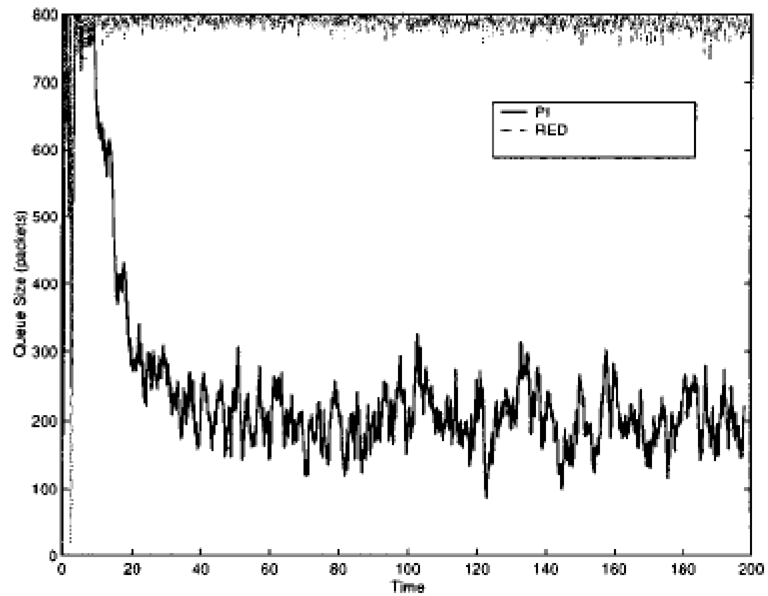
- C1=C2=15Mb/s, average packet size = 500 bytes, propagation delay range uniformly between 160ms and 240ms.
- Light TCP flow: 16 sessions, Heavy TCP flow: 400 sessions

Stability Methods – open/closed loop investigations

- Operating point was chosen as q_0=200 packets
- RED is generally slower than the PI-controller and it is not able to reach the desired OP for the heavy traffic situation



Light TCP flow



Heavy TCP flow

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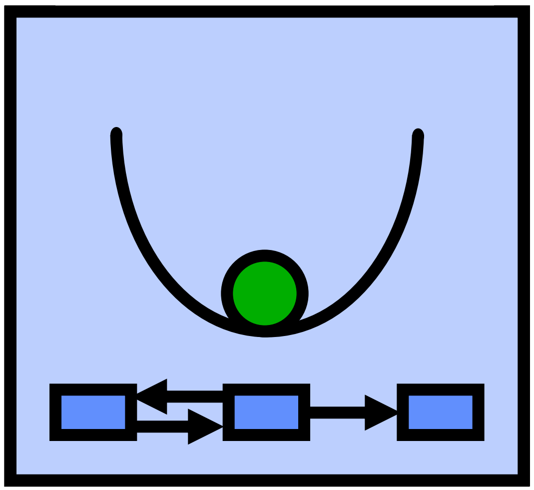

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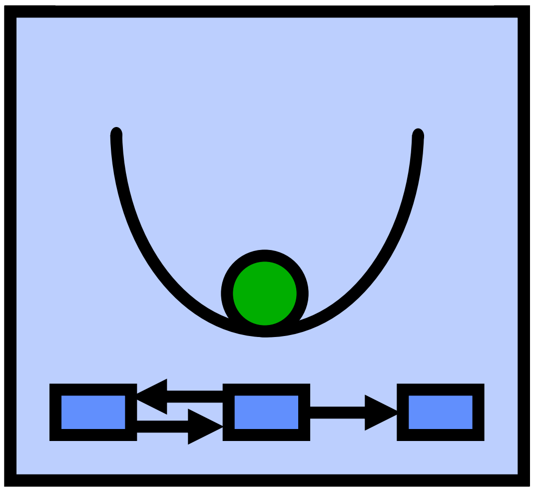
Conclusion

- Fluid models are capable of simulating the behavior of network algorithms
- Applying control laws known from automation engineering lead to a better understanding and a better performance of network algorithms
- Next step is to create a frame work for investigations of network control problems



Thank you for your attention!

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